Exercise 20

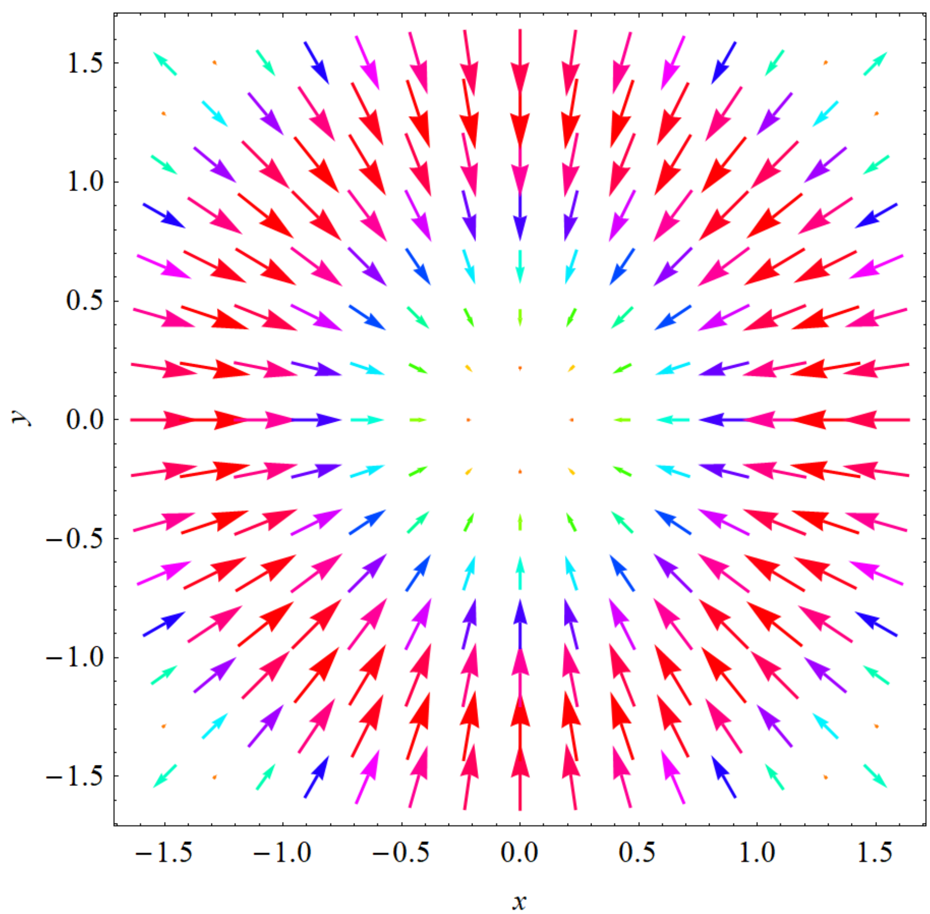
Let $\mathbf{F}(\mathbf{x}) = (r^2 - 2r)\mathbf{x}$, where $\mathbf{x} = \langle x, y\rangle$ and $r = |x|$. Use a CAS to plot this vector field in various domains until you can see what is happening. Describe the appearance of the plot and explain it by finding the points where $\mathbf{F}(\mathbf{x}) = \mathbf{0}$.

Solution

Start by rewriting the vector function so it's in terms of x and y only.

$$\begin{aligned}\mathbf{F}(x,y) &= (r^2 - 2r)\mathbf{x} \\ &= (r^2 - 2r)\langle x, y\rangle \\ &= \langle x(r^2 - 2r), y(r^2 - 2r)\rangle \\ &= \left\langle x\left[(x^2+y^2) - 2\sqrt{x^2+y^2}\right], y\left[(x^2+y^2) - 2\sqrt{x^2+y^2}\right]\right\rangle\end{aligned}$$

Using `VectorPlot` in Mathematica gives the following picture.



Inside the circle of radius 2, the vectors point radially inward; outside the circle of radius 2, the vectors point radially outward. $\mathbf{F}(\mathbf{x}) = \mathbf{0}$ when $r = 0$ or $r = 2$.